

function. In reality, it is forced to apply rather sophisticated mathematical techniques to obtain a very good approximation to the sine of 38 degrees, which it reports to you as the answer. The answer is no. A striking result from mathematics is that there are functions that are so complex that there is no well-defined, step-by-step process for determining their outputs based on their input values. A more powerful approach to computing functions is to follow directions provided by an algebraic formula rather than trying to display all possible input/output combinations in a table. If pressed to calculate the sine of 38 degrees, you might draw the appropriate triangle, measure its sides, and calculate the desired ratio—a process that cannot be expressed in terms of algebraic manipulations of the value 38. These functions are said to be noncomputable, whereas the functions whose output values can be determined algorithmically from their input values are said to be computable. We could, for example, use the algebraic formula  $V = P(1 + r)^n$  to describe how to compute the value of an investment of  $P$  after earning an annually compounded interest rate of  $r$  for  $n$  years. There are functions whose input/output relationships are too complex to be described by algebraic manipulations. Our question is whether we can always find a system for computing functions, regardless of their complexity. In turn, a fundamental task of computer science is to find techniques for computing the functions that lie beneath the problems we want to solve. An example is shown in Figure 12.1, which is an attempt to display the function that converts measurements in yards into equivalent measurements in meters. Because there is no limit to the list of possible input/output pairs, the table is destined to be incomplete. Examples include the .trigonometric functions such as sine and cosine