

A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. (We could also treat this as a fixed control volume that consists of the interior volume of the tank by disregarding the air that replaces the space vacated by the water.) This is obviously an unsteady-flow problem since the properties (such as the amount of mass) within the control volume change with time.

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Canceling the densities and other common terms and separating the variables give $dt = \frac{D_{\text{tank}}}{D_{\text{jet}}} \frac{dh}{\sqrt{2gh}}$. Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives $\int_0^t dt = \frac{D_{\text{tank}}}{D_{\text{jet}}} \int_{h_0}^{h_2} \frac{dh}{\sqrt{2gh}}$. Substituting, the time of discharge is determined to be $t = \frac{4 \text{ ft}}{\sqrt{32.2 \text{ ft/s}^2}} \left(\sqrt{h_0} - \sqrt{h_2} \right) = 0.5 \text{ in} \times \frac{1}{12 \text{ in}} \times 757 \text{ s} = 12.6 \text{ min}$. Therefore, it takes 12.6 min after the discharge hole is unplugged for half of the tank to be emptied. Discussion Using the same relation with $h_2 = 0$ gives $t = 43.1 \text{ min}$ for the discharge of the entire amount of water in the tank. Therefore, emptying the bottom half of the tank takes much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing h .

5–3 ? MECHANICAL ENERGY AND EFFICIENCY Many fluid systems are designed to transport a fluid from one location to another at a specified flow rate, velocity, and elevation difference, and the system may generate mechanical work in a turbine or it may consume mechanical work in a pump or fan during this process (Fig. 5–14). In Fig. 5–14, the mass flow rate of discharged water is $\dot{m}_{\text{out}} = \rho V A_{\text{jet}} \sqrt{2gh}$ (2) where $A_{\text{jet}} = \pi D_{\text{jet}}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is $m_{\text{CV}} = \rho V = \rho A_{\text{tank}} h$ (3) where $A_{\text{tank}} = \pi D_{\text{tank}}^2/4$ is the base area of the cylindrical tank. Substituting Eqs. 2 and 3 into the mass balance relation (Eq. 1) gives $2\rho \sqrt{2gh} A_{\text{jet}} = \frac{d(\rho A_{\text{tank}} h)}{dt} \Rightarrow 2\sqrt{2gh} (D_{\text{jet}}/D_{\text{tank}}) = \frac{dh}{dt}$. Water Air 0 $D_{\text{tank}} D_{\text{jet}} h_2 h_0 h$ FIGURE 5–13 Schematic for Example 5–2. Then the mechanical energy change of a fluid during incompressible flow becomes $\Delta e_{\text{mech}} = \frac{P_2}{\rho} - \frac{P_1}{\rho} + \frac{V_2^2}{2} - \frac{V_1^2}{2} + g(z_2 - z_1)$ (kJ/kg) (5–24). Therefore, the mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant. An ideal hydraulic turbine at the bottom elevation would produce the same work per unit mass $w_{\text{turbine}} = gh$ whether it receives water (or any other fluid with constant density) from the top or from the bottom of the container. Because of irreversibilities such as friction, mechanical energy cannot be converted entirely from one mechanical form to another, and the mechanical efficiency of a device or process is defined as $\eta_{\text{mech}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}}$. $E_{\text{mech, out}} = E_{\text{Flow work}}$ is expressed in terms of fluid properties, and it is convenient to view it as part of the energy of a flowing fluid and call it e_{mech} . FIGURE 5–14 Mechanical energy is a useful concept for flows that do not involve significant heat transfer or energy conversion, such as the flow of gasoline from an underground tank into a car. Therefore, the mechanical energy of a flowing fluid can be expressed on a unit-mass basis as $e_{\text{mech}} = \frac{P}{\rho} + \frac{V^2}{2} + gz$ where P/ρ is the flow energy, $V^2/2$ is the kinetic energy, and gz is the potential energy of the fluid, all per unit mass. The average velocity of the jet is approximated as $V = \sqrt{2gh}$, where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. The gravitational acceleration is 32.2 ft/s^2 .