

INTRODUCTION o In the last chapter, we considered electrostatic fields in free space or a space that has no materials in it. "vacuum" field theory o The formulas derived in Chapter 4 are still applicable, with some modification. o Electric fields can exist in free space, so as in material media. o Materials are broadly classified as conductors and non-conductors- insulators or dielectrics. o The electrical properties of materials :conduction, electric current, and polarization. o Further some properties of dielectric materials such as susceptibility, permittivity, linearity, isotropy, homogeneity, dielectric strength, and relaxation time. o The concept of boundary conditions for electric fields existing in two different media will be introduced. PROPERTIES OF MATERIALS o A discussion of the electrical properties of materials may seem out of place in a text of this kind. o But questions such as why an electron does not leave a conductor surface, why a current carrying wire remains uncharged, why materials behave differently in an electric field, and why waves travel with less speed in conductors than in dielectrics are easily answered by considering the electrical properties of materials. o A thorough discussion of this subject is usually found in texts on physical electronics or electrical engineering. o Here, a brief discussion will suffice to help us understand the mechanism by which materials influence an electric field. o In a broad sense, materials may be classified in terms of their conductivity  $\sigma$ , in mhos per meter or, more usually siemens per meter (S/m), as conductors and nonconductors, or technically as metals and insulators (or dielectrics). RESISTANCE AND CAPACITANCE o Using this equation, C can be obtained for any given two-conductor capacitance by following either of these methods o 1. Poisson's equation in those coordinate systems may be obtained by simply replacing zero on the right-hand side of the last three equations with  $-\rho$  o Laplace's equation is of primary importance in solving electrostatic problems involving a set of conductors maintained at different potentials. o If the space between the plates is filled with a homogeneous dielectric with permittivity  $\epsilon$  and we ignore flux fringing at the edges of the plates,  $D = \epsilon E$   $D = -\epsilon \frac{\partial V}{\partial x}$  Or  $E = -\frac{\partial V}{\partial x} = -\frac{Q}{\epsilon A}$  PARALLEL-PLATE CAPACITOR o and thus for a parallel-plate capacitor o This formula offers a means of measuring the dielectric constant  $\epsilon_r$ , of a given dielectric. UNIQUENESS THEOREM Since there are several methods (analytical, graphical, numerical, experimental, etc.) of solving a given problem, we may wonder whether solving Laplace's equation in different ways gives different solutions. o Also, a solution may be checked by going backward and finding out if it satisfies both Laplace's (or Poisson's) equation and the prescribed boundary conditions RESISTANCE AND CAPACITANCE o The problem of finding the resistance of a conductor of nonuniform cross section can be treated as a boundary-value problem. The resistance R (or conductance  $G = 1/R$ ) of a given conducting material can be found by following these steps: o Choose suitable coordinate system o Assume V as the potential difference between conductor terminals o Finally, Find  $R = V_0/I$  RESISTANCE AND CAPACITANCE o In essence, we assume  $V_0$ , find  $I$ , and determine  $R = V_0/I$ . ELECTROSTATIC BOUNDARY VALUE PROBLEMS o The procedure for determining the electric field E in the preceding chapters has generally been to use either Coulomb's law or Gauss's law when the charge distribution is known, or  $E = -\nabla V$  when the potential V is known throughout the region. o The values of conductivity of some common materials are shown in Table B. 1 in Appendix B. From this table, it is clear that materials such as copper and aluminum are metals, silicon and germanium are semiconductors, and glass and rubber are insulators. o Such problems are usually

tackled using Poisson's or Laplace's equation or the method of images, and they are usually referred to as boundary value problems. equation can be found that satisfies the o Similar steps can be taken to show that the theorem applies to Poisson's equation and to prove the theorem for the case where the electric field (potential gradient) is specified on the boundary. Suppose that each of the plates has an area  $S$  and they are separated by a distance  $d$ . o We assume that plates 1 and 2, respectively, carry charges  $+Q$  and  $-Q$  uniformly distributed on them so that  $\rho_s = Q/S$  PARALLEL-PLATE CAPACITOR o An ideal parallel-plate capacitor is one in which the plate separation  $d$  is very small compared with the dimensions of the plate. o Dielectric materials have few electrons available for conduction of current, whereas metals have an abundance of free electrons. POISSON'S AND LAPLACE'S EQUATIONS Poisson's and Laplace's equations are easily derived from Gauss's law (for a linear material medium):  $\nabla \cdot \mathbf{D} = \rho_v$ .  $\mathbf{D} = \epsilon \mathbf{E}$ .  $\mathbf{E} = -\nabla V$ . Thus, any solution of Laplace's equation which satisfies the same boundary conditions must be the only solution regardless of the method used. GENERAL PROCEDURES FOR SOLVING POISSON'S OR LAPLACE'S EQUATION The following general procedure may be taken in solving a given boundary-value problem involving Poisson's or Laplace's equation: 1. o Alternatively, it is possible to assume current  $I$ , find the corresponding potential difference  $V$ , and determine  $R$  from  $R = V/I$  o As will be discussed shortly, the capacitance of a capacitor is obtained using a similar technique. At temperatures near absolute zero ( $T = 0 \text{ K}$ ), some conductors exhibit infinite conductivity and are called superconductors. =  $\rho_s = Q/S$ , or  $V(\infty, 0, \infty)$ . 1.  $\rho_s = Q/S$