

Forecasting of asset market prices is of interest from a practical and theoretical perspective. Besides the fixed part of various initial parameters, the dynamic part of the pool is updated by adding successful parameters so that we keep a pool of various and most recently used candidate parameters. If one can determine a set of parameters within the computational and time constraints to make an accurate forecast, then it is generally not desirable to reduce the number of parameters considerably in order to obtain a forecast that is almost as good. Our approach uses the asset flow differential equations (AFDE) that have been developed by Caginalp and collaborators since 1989 (see Caginalp and Balenovich [11] and references contained therein). It utilizes a basic supply and demand adjustment equation, but also incorporates the finiteness of assets, rather than the assumption of infinite arbitrage capital of classical finance theory. The fact that these funds often trade at significant discounts (e.g. 5% to 25%) and sometimes at large premiums (e.g. 50%) has been a puzzle to classical finance, and many papers have addressed these issues (see Anderson and Born for a summary [2]). A number of these papers have focussed on reconciling these discounts to the efficient market hypothesis (EMH) whose centerpiece is the concept that the market trading price should reflect all available public information and should therefore reflect the true value. Thus, if one hypothesizes a price trend motivation, the confirmation is the determination of a coefficient that is positive and statistically significant. The implementation of these differential equations for practical forecasts poses challenging mathematical tasks that are inverse problems. We perform this optimization by a nonlinear computational algorithm that combines a quasi-Newton weak line search with the BFGS formula. We select an initial parameter vector from the initial parameter pool because the optimization success of quasi-Newton method in the algorithm depends on the initial parameter. In this paper, we utilize a system of differential equations that incorporate the valuation and trend motivations similar to Caginalp and Balenovich [11]. Briefly, a closed-end fund (see Bodie et al. [6]) is formed as investors pool a sum of money for a particular investment, e.g., investing in common stocks in Japan. One of these consists of purely statistical methods, e.g., time series, that strive to uncover a statistically significant pattern in the data. A second involves developing some understanding of the underlying processes and deriving, for example, differential equations. The alternative, however, is to utilize physical laws and estimate some parameters statistically whereupon the differential equations can be used to make a forecast. However, the issue of the origin of the model is less relevant when it is possible to perform out-of-sample forecast (as is the case for weather forecasting and stock price forecasting, for example) that can be tested statistically to determine the accuracy of the predictions. The equations can readily incorporate additional motivational aspects of trading as they are established. In fact, one way of implementing this is to modify the differential equations to difference equations, and then to use statistical methods to evaluate the coefficients. We use nonlinear least-square technique with initial value problem approach by focusing on the market price variable  $P$  since any real data for the other three variables  $B$ ,  $\beta_1$  and  $\beta_2$  in the dynamical system is not available explicitly. Here, the gradient  $(\nabla F(x))$  is approximated by using the central difference formula and step length  $s$  is determined by the backtracking line search among several choices in literature (see Nocedal and Wright [29]). Nevertheless, the methodology for developing a valuation model is well established in finance, and a model for valuation can be constructed for a particular asset class. Once a

set of parameters characterizing an investor population is specified, the differential equations can be solved for future times. We construct a pool of initial parameters  $K_i$  chosen via a set of grid points in a hyper-box.