

This chapter defines algebraic structures, starting with internal composition laws on a set G , illustrated by addition and multiplication on number sets and composition of maps. Associative, commutative, neutral, and inverse elements are defined, leading to the definition of a group $(G, \cdot)$ requiring associativity, a neutral element, and inverses for all elements. Commutative groups are also defined.

Key theorems establish the uniqueness of the neutral and inverse elements, and a formula for the inverse of a product. Subgroups H of G are defined, characterized by non-emptiness, closure under the operation and inverses. Equivalent conditions for subgroups are provided, with specific examples for additive and multiplicative groups. The chapter then introduces rings $(A, +, \cdot)$, defined as a commutative group under $(+)$ and an associative operation $(\cdot)$ distributive over $(+)$. Commutative rings and rings with identity are defined. Subrings are defined and characterized. Finally, fields $(\mathbb{F}, +, \cdot)$ are defined as rings where all non-zero elements are invertible; commutative fields are also defined. Subfields are characterized as subrings where inverses of non-zero elements exist, with examples showing $(\mathbb{Q}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$, and $(\mathbb{C}, +, \cdot)$ as subfields of $(\mathbb{C}, +, \cdot)$.