Chapter 1Oscillatory MotionPeriodic motion is motion of an object that regularly repeats—the object returns to a given position after a fixed time interval. With a little thought, we can identify several types of periodic motion in everyday life. Your car returns to the driveway each after-noon. You return to the dinner table each night to eat. A bumped chandelier swings back and forth, returning to the same position at a regular rate. The Earth returns to the same position in its orbit around the Sun each year, resulting in the variation among the four seasons. The Moon returns to the same relationship with the Earth and the Sun, resulting in a full Moon approximately once a month. In addition to these everyday examples, numerous other systems exhibit periodic motion. For example, the molecules in a solid oscillate about their equilibrium positions; electromagnetic waves, such as light waves, radar, and radio waves, are characterized by oscillating electric and magnetic field vectors; and in alternating-current electrical circuits, voltage, current, and electric charge vary periodically with time.A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position of the object relative to some equilibrium position. If this force is always directed toward the equilibrium position, the motion is called simple harmonic motion, which is the primary focus of this chapter.1.1 Motion of an Object Attached to a SpringAs a model for simple harmonic motion, consider a block of mass mattached to the end of a spring, with the block free to move on a horizontal, frictionless surface(Fig.1.1). When the spring is neither stretched nor compressed, the block is at theposition called the equilibrium positionof the system, which we identify as x = 0. Weknow from experience that such a system oscillates back and forth if disturbed from itsequilibrium position.We can understand the motion in Figure 1.1 qualitatively by first recalling thatwhen the block is displaced to a position x, the spring exerts on the block a force thatis proportional to the position and given by Hooke's lawFs = □kx (1.1)We call this a restoring force because it is always directed toward the equilibrium position and therefore opposite the displacement from equilibrium. That is, when the block is displaced to the right of x= 0 in Figure 1.1, then the position is positive and 1the restoring force is directed to the left. When the block is displaced to the left of x= 0, then the position is negative and the restoring force is directed to the right.Applying Newton's second law □Fx = maxto the motion of the block, with Equation 1.1 providing the net force in the x direction, we obtain□kx = maxax□ □ kmx(1.2)Figure 1.1 A block attached to a spring moving on a frictionless surface. (a) When the block is displaced to the right of equilibrium (x>0), the force exerted by the spring acts to the left. (b) When the block is at its equilibrium position (x=0), the force exerted by the spring is zero.(c) When the block is displaced to the left of equilibrium (x Systems that behave inthis way are said to exhibit simple harmonic motion. An object moves with simpleharmonic motion whenever its acceleration is proportional to its position and isoppositely directed to the displacement from equilibrium.If the block in Figure 1.1 is displaced to a position x= A and released from rest,its initial acceleration is □kA/m. When the block passes through the equilibrium position x= 0, its acceleration is zero. At this instant, its speed is a maximum because the acceleration changes sign. The block then continues to travel to the left of equilibrium with a positive acceleration and finally reaches x= □A, at which time its acceleration is +kA/m and its speed is again zero. The block completes a full cycle of 2its motion by returning to the original position, again passing through x= 0 with maximum speed. Thus, we see that the block oscillates between the turning points x=□A. In the absence of friction, because the force exerted by the spring is conservative, this idealized motion will continue forever. Real systems are generally subject to friction, so they do not oscillate forever. We explore the details of thesituation with friction in Section 1.6.As Pitfall Prevention 1.1 points out, the principles that we develop in this chapter are also valid for an object hanging from a vertical spring, as long as we recognize that the weight of the object will stretch the spring to a new equilibrium position x= 0. To prove this statement, let xs represent the total extension of the spring from its equilibrium position without the hanging object. Then, xs = (mg/k)+x, where (mg/k) is the extension of the spring due to the weight of the hanging object and x is the instantaneous extension of the spring due to the simple harmonic motion. The magnitude of the net force on the object is then Fs Fg = k ((mg/k)+x) mg = kx. The net force on the object is the same as that on a block connected to a horizontal spring as in Equation 1.1, so the same simple harmonic motion results.1.2 Mathematical Representation of Simple Harmonic MotionLet us now develop a mathematical representation of the motion we described in the preceding section. We model the block as a particle subject to the force in Equation 1.1. We will generally choose x as the axis along which the oscillation occurs; hence, we will drop the subscript-x notation in this discussion. Recall that, by definition, a = dv/dt = d2x/dt2, and so we can express Equation 1.2 as2d x2dt kmx(1.3)If we denote the ratio k/m with the symbol 2(we choose 2rather than 'in order tomake the solution that we develop below simpler in form), then2k (1.4)mand Equation 1.3 can be written in the form2d x 2 x(1.5)2dt3What we now require is a mathematical solution to Equation 1.5—that is, a function x(t) that satisfies this second-order differential equation. This is a mathematical representation of the position of the particle as a function of time. We seek a function x(t) whose second derivative is the same as the original function with a negative sign and multiplied by 2. The trigonometric functions sine and cosine exhibit this behavior, so we can build a solution around one or both of these. The following cosine function is a solution to the differential equation:x(t) = A cos (t +) (1.6)where A, , and (are constants. To see explicitly that this equation satisfies Equation1.5, note thatdxdt Addtcos(t) Asin (t)(1.7)2d x Ad22dtdtsin(t A) cos(t)(1.8)Comparing Equations 1.6 and 1.8, we see that d2x/dt2= 2x and Equation 1.5 issatisfied.he parameters A, , and are constants of the motion. In order to give physicalsignificance to these constants, it is convenient to form a graphical representation ofthe motion by plotting x as a function of t, as in Figure 1.2a. First, note that A, calledthe amplitude of the motion, is simply the maximum value of the position of theparticle in either the positive or negative x direction. The constant 'is called the angular frequency, and has units of rad/s. It is a measure of how rapidly the oscillations are occurring—the more oscillations per unit time, the higher is the value of . From Equation 1.4, the angular frequency is km(1.9)The constant angle (is called the phase constant(or initial phase angle) and,along with the amplitude A, is determined uniquely by the position and velocity of the particle at t = 0. If the particle is at its maximum position x=A at t = 0, the phase constant is = 0 and the graphical representation of the motion is shown in Figure 1.2b. The quantity (t +) is called the phase of the motion. Note that the function x(t) is periodic and its value is the same each time t increases by 2 radians.4Equations 1.1, 1.5, and 1.6 form the basis of the mathematical representation of simple harmonic motion. If we are analyzing a situation and find that the force on a particle is of the mathematical form of Equation 1.1, we know that the motion will be that of a simple harmonic oscillator and that the position of the particle is described by Equation 1.6. If we analyze a system and find that it is described by a differential equation of the form of Equation 1.5, the motion will be that of a simple harmonic oscillator. If we analyze a situation and find that the position of a particle is described by Equation 1.6, we know the particle is undergoing simple harmonic motion.Figure 1.2(a) An x–vs.–t graph for an object undergoing simple harmonic motion. The amplitude of the motion is A, the period is T, and the phase constant is □. (b) The x–vs.–tgraph in the special case in which x=A at t = 0 and hence □ = 0.An experimental arrangement that exhibits simple harmonic motion is illustrated in Figure 1.3. An object oscillating vertically on a spring has a pen attached to it. While the object is oscillating, a sheet of paper is moved perpendicular to the direction of motion of the spring, and the pen traces out the cosine curve in Equation 1.6.5Figure 1.3 An experimental apparatus for demonstrating simple harmonic motion. A pen attached to the oscillating object traces out a sinusoidal pattern on the moving chart paper.Figure 1.4An x–t graph for an object undergoing simple harmonic motion. At a particular time, the object's position is indicated by (A) in the graph.Figure 11.5 Two x–t graphs for objects undergoing simple harmonic motion. The amplitudes and frequencies are different for the two objects.6Let us investigate further the mathematical description of simple harmonic motion. The period T of the motion is the time interval required for the particle to go through one full cycle of its motion (Fig. 1.2a). That is, the values of x and v for the particle at time t equal the values of x and v at time t + T. We can relate the period to the angular frequency by using the fact that the phase increases by 2□ radians in a time interval of T:[□(t + T) + □] □ (□t + □) = 2□Simplifying this expression, we see that □T=2□, orT□2□□(1.10)The inverse of the period is called the frequency f of the motion. Whereas the period is the time interval per oscillation, the frequency represents the number of oscillations that the particle undergoes per unit time interval:f□ T1□2□□The units of f are cycles per second, or hertz(Hz). Rearranging Equation 1.11 gives□ □ 2□ f□2□T(1.12)We can use Equations 1.9, 1.10, and 1.11 to express the period and frequency of the motion for the particle–spring system in terms of the characteristics m and k of the system asT □ 2□□ 2□□mk(1.13)f□ 1 T□12□km(1.14)That is, the period and frequency depend only on the mass of the particle and the force constant of the spring, and not on the parameters of the motion, such as A or □. As we might expect, the frequency is larger for a stiffer spring (larger value of k) anddecreases with increasing mass of the particle.We can obtain the velocity and acceleration of a particle undergoing simple harmonic motion from Equations 1.7 and 1.8:7vdxdt□ □□Asin (□t □ □)(1.15)a2d x2dt2□ □□ A sin (t □ □ □)(1.16)From Equation 1.15 we see that, because the sine and cosine functions oscillatebetween □1, the extreme values of the velocity v are □ □A. Likewise, Equation 1.16tells us that the extreme values of the acceleration a are □ □2A. Therefore, the maximum values of the magnitudes of the velocity and acceleration arevmax□ □A □kAm(1.17)a max2□ □ A □kAm(1.18)Figure 1.6a plots position versus time for an arbitrary value of the phase constant. The associated velocity–time and acceleration–time curves are illustrated in Figures 1.6b and 1.6c. They show that the phase of the velocity differs from the phase of the position by □/2 rad, or 90°. That is, when x is a maximum or a minimum, the velocity is zero. Likewise, when x is zero, the speed is a maximum. Furthermore, note that the phase of the acceleration differs from the phase of the position by □ radians, or 180°. For example, when x is a maximum, a has a maximum magnitude in the opposite direction.8Figure 1.6 Graphical representation of simple harmonic motion. (a) Position versus time. (b) Velocity versus time. (c) Acceleration versus time. Note that at any specified time the velocity is 90°out of phase with the position and the acceleration is 180°out of phase with the position.Equation 1.6 describes simple harmonic motion of a particle in general. Let usnow see how to evaluate the constants of the motion. The angular frequency is evaluted using Equation 1.9. The constants A and (are evaluated from the initial conditions, that is, the state of the oscillator at t = 0.Suppose we initiate the motion by pulling the particle from equilibrium by a distance A and releasing it from rest at t = 0, as in Figure 1.7. We must then require thatFigure 1.7 A block–spring system that begins its motion from rest with the block at x=A at t = 0. In this case, ￼ = 0 and thus x= A cos ￼t.9Figure 1.8 (a) Position, velocity, and acceleration versus time for a block undergoing simple harmonic motion under the initial conditions that at t= 0, x(0)= A and v(0) = 0. (b) Position, velocity, and acceleration versus time for a block undergoing simple harmonic motion under the initial conditions that at t= 0, x(0) = 0 and v (0)= vi.our solutions for x(t) and v(t) (Eqs. 1.6 and 1.15) obey the initial conditions that x(0) = A and v(0) = 0:x(0) = A cos ￼ = Av(0) = ￼￼A sin ￼ = 0These conditions are met if we choose ￼ = 0, giving x = A cos ￼t as our solution. Tocheck this solution, note that it satisfies the condition that x(0)= A, because cos 0= 1.The position, velocity, and acceleration versus time are plotted in Figure 1.8a for this special case. The acceleration reaches extreme values of￼￼2A when the position has extreme values of ￼A. Furthermore, the velocity has extreme values of￼￼A, which both occur at x= 0. Hence, the quantitative solution agrees with our qualitative description of this system.Let us consider another possibility. Suppose that the system is oscillating and we define t = 0 as the instant that the particle passes through the unstretched positionof the spring while moving to the right (Fig. 1.9). In this case we must require that our solutions for x(t) and v(t) obey the initial conditions that x(0) = 0 and v(0) = vi:x(0) = A cos ￼ = 010v(0) = ￼￼A sin ￼ = viThe first of these conditions tells us that ￼ = ￼ ￼/2. With these choices for ￼, the second condition tells us that A = ￼vi/￼. Because the initial velocity is positive and the amplitude must be positive, we must have ￼ = ￼￼/2. Hence, the solution is given byx￼vi￼cos ￼￼ ￼￼ ￼t￼2￼￼￼The graphs of position, velocity, and acceleration versus time for this choice of t = 0 are shown in Figure 1.8b. Note that these curves are the same as those in Figure 1.8a, but shifted to the right by one fourth of a cycle. This is described mathematically by the phase constant ￼ = ￼￼/2, which is one fourth of a full cycle of 2￼.Figure 1.9 The block–spring system is undergoing oscillation, and t = 0 is defined at an instant when the block passes through the equilibrium position x= 0 and is moving to the right with speed vi.Example 1.1 An Oscillating ObjectAn object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation where t is in seconds and the angles in the parentheses are in radians.(A) Determine the amplitude, frequency, and period of the motion.Solution: By comparing this equation with Equation 1.6, x = A cos (￼t + ￼), we see that A = 4.00 m and ￼ rad/s. Therefore, f = ￼/2￼ = 0.500 Hz and T = 1/f = 2.00 s.(B) Calculate the velocity and acceleration of the object at any time t.Solution Differentiating x to find v, and v to find a, we obtaindt￼ ￼ (4.00 m/s)sin ￼￼ ￼￼ ￼t ￼4￼￼￼ dtd(￼t)dxv ￼ 11￼ ￼ (4.00 ￼ m/s)sin (t￼ ￼￼4)dt￼ ￼ (4.00 ￼ m/s)cos ￼￼ ￼￼ ￼t dvv ￼ ￼ ￼ (4.00 ￼ 2 2 m/s) cos(t￼ ￼￼4)￼4￼￼￼ ddt(￼t)(C) Using the results of part (B), determine the position, velocity, and acceleration of the object at t = 1.00s.Solution Noting that the angles in the trigonometric functions are in radians, we obtain, at t

=1.00 s.x (4.00 m) cos   4 (4.00m) cos 54   (4.00m)(0.707) 2.83mv   (4.00 m/s)sin 54   (4.00m/s)(0.707) 8.89m/s a (4.00 2 2m/s) cos 54   (4.00 2 2 m/s)(0.707) 27.9m/s2(D) Determine the maximum speed and maximum acceleration of the object.Solution In the general expressions for v and a found in part (B), we use the fact that the maximum values of the sine and cosine functions are unity. Therefore, v varies between 4.00 m/s, and a varies between 4.00 2 m/s2.Thus,vmax = 4.00 m/s = 12.6 m/samax = 4.002 m/s2= 39.5 m/s2We obtain the same results using the relations vmax = A and amax = 2A, where A = 4.00m and = rad/s.(E) Find the displacement of the object between t = 0 and t = 1.00s.12Solution The position at t = 0 isIn part (C), we found that the position at t = 1.00s is −2.83m; therefore, the displacement between t = 0 and t = 1.00s isx = xf xi = 2.83 m 2.83 m = 5.66 mBecause the object's velocity changes sign during the first second, the magnitude of x is not the same as the distance traveled in the first second. (By the time the first second is over, the object has been through the point x = 2.83m once, traveled to x = 4.00m, and come back to x = "2.83m.)Example 1.2 Watch Out for Potholes!A car with a mass of 1 300kg is constructed so that its frame is supported by four springs. Each spring has a force constant of 20 000N/m. If two people riding in the car have a combined mass of 160kg, find the frequency of vibration of the car after it is driven over a pothole in the road.Solution To conceptualize this problem, think about your experiences with automobiles. When you sit in a car, it moves downward a small distance because your weight is compressing the springs further. If you push down on the front bumper and release, the front of the car oscillates a couple of times. We can model the car as being supported by a single spring and categorize this as an oscillation problem based on our simple spring model. To analyze the problem, we first need to consider the effective spring constant of the four springs combined. For a given extension x of the springs, the combined force on the car is the sum of the forces from the individual springs:F total (kx) kx where x has been factored from the sum because it is the same for all four springs. We see that the effective spring constant for the combined springs is the sum of the individual spring constants:keff = k = 4 x 20 000 N/m = 80 000 N/mHence, the frequency of vibration is, from Equation 1.14,f1 2keff m1280 000 N/m1460 kg1.18 HzTo finalize the problem, note that the mass we used here is that of the car plus the people, because this is the total mass that is oscillating. Also note that we have explored only up-and-down motion of the car. If an oscillation is established in which 13the car rocks back and forth such that the front end goes up when the back end goes down, the frequency will be different.What If? Suppose the two people exit the car on the side of the road. One of them pushes downward on the car and releases it so that it oscillates vertically. Is the frequency of the oscillation the same as the value we just calculated?Answer The suspension system of the car is the same, but the mass that is oscillating is smaller—it no longer includes the mass of the two people. Thus, the frequency should be higher. Let us calculate the new frequency:f1 2keff m1280 000 N/m1300 kg1.25 HzAs we predicted conceptually, the frequency is a bit higher.Example 1.3 A Block–Spring SystemA 200-g block connected to a light spring for which the force constant is 5.00N/m is free to oscillate on a horizontal, frictionless surface. The block is displaced 5.00cm from equilibrium and released from rest, as in Figure 1.7.(A) Find the period of its motion.Solution: From Equations 1.9 and 1.10, we know that the angular frequency of a block–spring system is and the period isk5.00 N/m =

5.00 rad/sm−3kg200 10 and the period isT 2 2 5.00 rad/s(B) Determine the maximum speed of the block.Solution: We use Equation 1.17: 1.26 svmax = A = (5.00 rad/s) (5.00 10−2 m) = 0.250 m/s(C) What is the maximum acceleration of the block?Solution: We use Equation 1.18:vmax = 2A = (5.00 rad/s)2(5.00 10−2 m) = 1.25 m/s214(D) Express the position, speed, and acceleration as functions of time.Solution: We find the phase constant from the initial condition that x = A at t = 0:x(0) = A cos = Awhich tells us that = 0. Thus, our solution is x= A cos t. Using this expression and the results from (A), (B), and (C), we find thatx = A cos t = (0.050 m/s) cos 5.00tv = A cos t = a = − 2A cos t = − (0.250 m/s) sin 5.00t− (1.25 m/s2) cos 5.00tWhat If? What if the block is released from the same initial position, xi = 5.00 cm, but with an initial velocity of vi = 0.100 m/s? Which parts of the solution change andwhat are the new answers for those that do change?Answers: Part (A) does not change—the period is independent of how the oscillator is set into motion. Parts (B), (C), and (D) will change. We begin by considering position and velocity expressions for the initial conditions:(1) x(0) = A cos = xi(2) v(0) = − A sin = viDividing Equation (2) by Equation (1) gives us the phase constant: Asin viAcos xitan = vi xi = 0.12 0.100 m(5.00 rad/s)(0.0500m) 0.400Now, Equation (1) allows us to find A:A xicos 0.050 0mcos(0.12) 0.0539mThe new maximum speed isvmax = A = (5.00 rad/s) (5.39 10−2 m) = 0.269 m/sThe new magnitude of the maximum acceleration isamax = 2A = (5.00 rad/s)2(5.39 10−2 m) = 1.35 m/s215The new expressions for position, velocity, and acceleration arex = (0.053 9m) cos (5.00t + 0.12)v = −(0.269 m/s) sin (5.00t + 0.12)a = −(1.35 m/s2) cos (5.00t + 0.12)As we saw in Chapters 7 and 8, many problems are easier to solve with an energy approach rather than one based on variables of motion. This particular What If? is easier to solve from an energy approach. Therefore, in the next section we shall investigate the energy of the simple harmonic oscillator.1.3 Energy of the Simple Harmonic OscillatorLet us examine the mechanical energy of the block–spring system illustrated in Figure 1.1. Because the surface is frictionless, we expect the total mechanical energy of the system to be constant. We assume a mass less spring, so the kinetic energy of the system corresponds only to that of the block. We can use Equation 1.15 to express the kinetic energy of the block asK 1 2mv 2 122 m A 2 2sin (t)(1.19)The elastic potential energy stored in the spring for any elongation x is given by 21kx. Using Equation 1.6, we obtain2211 2 2 kA cos (U t)kx 22(1.20)We see that K and U are always positive quantities. Because 2k/m, we can expressthe total mechanical energy of the simple harmonic oscillator asE K U 1 t) cos (2 22 k A 2[sin (t)] From the identity sin2 cos2 = 1, we see that the quantity in square brackets is unity. Therefore, this equation reduces to1 kAE 22(1.21)That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude. Note that U is small when Kis large, and vice versa, because the sum must be constant. In fact, the total mechanical energy is equal to the maximum potential energy stored in the spring when x= Abecause v = 0 at these points and thus there is no kinetic energy. At the equilibrium 16position, where U = 0 because x = 0, the total energy, all in the form of kinetic energy, is again221kA. That is,E 1 2mv 2 max 122 m A 2 12m kA m2 12k A2(at x = 0)Plots of the kinetic and potential energies versus time appear in Figure 1.10a,where we have taken = 0. As already mentioned, both K and U are always positive,and at all times their sum is a constant equal to 21kA, the total energy of the system.2The variations of K and U with the position x of the block are plotted in Figure 1.10b.Figure 1.10(a) Kinetic energy and potential energy versus time for a simple harmonic oscillator with = 0. (b) Kinetic energy and potential energy versus position for a simple harmonic oscillator. In either plot, note that K + U = constant.Energy is continuously being transformed between potential energy stored in thespring and kinetic energy of the block.Figure 1.11 illustrates the position, velocity, acceleration, kinetic energy, and potential energy of the block–spring system for one full period of the motion. Most of the ideas discussed so far are incorporated in this important figure. Study it carefully.Finally, we can use the principle of conservation of energy to obtain the velocity for an arbitrary position by expressing the total energy at some arbitrary position x as17E K U 1 2mv 2 12kx 2 12kA 2v   k(A m2 2 x)   A 2 x2(1.22)When we check Equation 1.22 to see whether it agrees with known cases, we find thatit verifies the fact that the speed is a maximum at x = 0 and is zero at the turning points x= A.You may wonder why we are spending so much time studying simple harmonic os-cillators. We do so because they are good models of a wide variety of physical phenomena. For example, recall the Lennard–Jones potential. This complicated function describes the forces holding atoms together. Figure 1.12a shows that, for small displacements from the equilibrium position, the potential energy curve for this function approximates a parabola, which represents the potential energy function for a simple harmonic oscillator. Thus, we can model the complex atomic binding forces as being due to tiny springs, as depicted in Figure 1.12b.Figure 1.11 Simple harmonic motion for a block–spring system and its analogy to the motion of a simple pendulum (Section 1.5). The parameters in the table at the right refer to the block–spring system, assuming that at t = 0, x = A so that x = A cos t.18 Figure 1.12(a) If the atoms in a molecule do not move too far from their equilibrium positions, a graph of potential energy versus separation distance between atoms is similar to the graph of potential energy versus position for a simple harmonic oscillator (blue curve). (b) The forces between atoms in a solid can be modeled by imagining springs between neighboring atoms.The ideas presented in this chapter apply not only to block–spring systems andatoms, but also to a wide range of situations that include bungee jumping, tuning in atelevision station, and viewing the light emitted by a laser. You will see more examples of simple harmonic oscillators as you work through this book.Example 1.4 Oscillations on a Horizontal SurfaceA 0.500-kg cart connected to a light spring for which the force constant is 20.0N/m oscillates on a horizontal, frictionless air track.(A) Calculate the total energy of the system and the maxi-mum speed of the cart if the amplitude of the motion is 3.00cm.Solution: Using Equation 1.21, we obtain2 1E 1k A (20.0 N/m)(3.0010 22-2 m)2= 9.00 10-3J When the cart is located at x = 0, we know that U = 0 and E = 1 mv22max = 9.00 10-3Jvmax2(9.00 10 30.500 kgJ) 0.190 m/s191 mv22max ; therefore(B) What is the velocity of the cart when the position is 2.00cm?Solution: We can apply Equation 1.22 directlyv   km(A 2 x2)   20.0 N/m 0.500 kg = 0.141 m/s[(0.0300 m) 2 2(0.0200 m)]The positive and negative signs indicate that the cart could be moving to either the right or the left at this instant.(C) Compute the kinetic and potential energies of the sys-tem when the position is 2.00cm.Solution: Using the result of (B), we find thatK mv 2 1 1(0.500 kg) (0.141m/s) 222 5.00 10 -3JU 1 2k x 2 Note that K + U = E.12(20.0 N/m)(0.0200m) 2 4.00 10 -3JWhat If? The motion of the cart in this example could have been initiated by releasing the cart from rest at x!3.00cm. What if the cart were released from the same position, but with an initial velocity of v = -0.100m/s? What are the new amplitude and maximum speed of the cart?Answer: This is the same type of question as we asked at the end of Example 1.3, but here we apply an energy approach. First let us calculate the total energy of the system at t = 0, which consists of both kinetic energy and potential energy:E � mv � 1 1kx 22 22�1(0.500 kg)(-0.100m/s) 2= 1.5 � 10-2J2 � 12(20.0 N/m)(0.0300 m)2To find the new amplitude, we equate this total energy to the potential energy when the cart is at the end point of the motion:21 kAE � 220A� 2 Ek�2(1.15 �10 �220.0 N/mJ) �0.0339 mNote that this is larger than the previous amplitude of 0.0300 m. To find the new maximum speed, we equate this total energy to the kinetic energy when the cart is at the equilibrium position:21 E � mv max 2vmax� 2 Ek�2(1.15 �10 �20.500 N/mJ) �0.214 m/sThis is larger than the value found in part (a) as expected because the cart has an initial velocity at t = 0.1.4 Comparing Simple Harmonic Motion with Uniform Circular MotionSome common devices in our everyday life exhibit a relationship between oscillatory motion and circular motion. For example, the pistons in an automobile engine (Figure 1.13a) go up and down—oscillatory motion—yet the net result of this motion is circular motion of the wheels. In an old-fashioned locomotive (Figure 1.13b), the drive shaft goes back and forth in oscillatory motion, causing a circular motion of the wheels. In this section, we explore this interesting relationship between these two types of motion. We shall use this relationship again when we study electromagnetism and when we explore optics.Figure 1.14 is an overhead view of an experimental arrangement that shows thisrelationship. A ball is attached to the rim of a turntable of radius A, which is illuminated from the side by a lamp. The ball casts a shadow on a screen. We find that as the turntable rotates with constant angular speed, the shadow of the ball moves back and forth in simple harmonic motion.21Figure 1.13(a) The pistons of an automobile engine move in periodic motion along a single dimension. This photograph shows a cutaway view of two of these pistons. This motion is converted to circular motion of the crankshaft, at the lower right, and ultimately of the wheels of the automobile. (b)The back-and-forth motion of pistons (in the curved housing at the left) in an old-fashioned locomotive is converted to circular motion of the wheels.22Figure 1.14 An experimental setup for demonstrating the connection between simple harmonic motion and uniform circular motion. As the ball rotates on the turntable with constant angular speed, its shadow on the screen moves back and forth in simple harmonic motion.Figure 1.15. Relationship between the uniform circular motion of a point P and the simple harmonic motion of a point Q. A particle at P moves in a circle of radius A with constant angular speed �. (a) A reference circle showing the position of P at t = 0. (b)The xcoordinates of points P and Q are equal and vary in time according to the expression x= A cos (�t + �). (c) The x component of the velocity of P equals the velocity of Q. (d) The xcomponent of the acceleration of P equals the acceleration of Q.Consider a particle located at point P on the circumference of a circle of radius A, as in Figure 1.15a, with the line OP making an angle (with the x axis at t = 0. We call this circle a reference circle for comparing simple harmonic motion with uniform 23circular motion, and we take the position of P at t = 0 as our reference position. If the particle moves along the circle with constant angular speed 'until OP makes an angle with the x axis, as in Figure 1.15b, then at some time t0, the angle between OP and the x axis is � = �t + �. As the particle moves along the circle, the projection of P on the x axis, labeled point Q, moves back and forth along the x axis between the limits x= �A.Note that points P and

Q always have the same x coordinate. From the right triangle OPQ, we see that this x coordinate isx (t) = A cos (⍰t + ⍰) (1.23)This expression is the same as Equation 1.6 and shows that the point Q moves with simple harmonic motion along the x axis. Therefore, we conclude thatWe can make a similar argument by noting from Figure 1.15b that the projection of P along the y axis also exhibits simple harmonic motion. Therefore, uniform circular motion can be considered a combination of two simple harmonic motions, one along the x axis and one along the y axis, with the two differing in phase by 90°.This geometric interpretation shows that the time interval for one complete revolution of the point P on the reference circle is equal to the period of motion T for simple harmonic motion between x = ⍰ A. That is, the angular speed 'of P is the same as the angular frequency 'of simple harmonic motion along the x axis. (This is why we use the same symbol.) The phase constant ⍰ for simple harmonic motion corresponds to the initial angle that OP makes with the x axis. The radius A of the reference circleequals the amplitude of the simple harmonic motion.Because the relationship between linear and angular speed for circular motion isv = r⍰, the particle moving on the reference circle of radius A has a velocity of magnitude ⍰A. From the geometry in Figure 1.15c, we see that the x component of this velocity is –⍰A sin (⍰t + ⍰). By definition, point Q has a velocity given by dx/dt. Differentiating Equation 1.23 with respect to time, we find that the velocity of Q is the same as the x component of the velocity of P.The acceleration of P on the reference circle is directed radially inward toward O and has a magnitude v2/A = ⍰2A. From the geometry in Figure 1.15d, we see that thex component of this acceleration is –⍰2A cos (⍰t + ⍰). This value is also the acceleration of the projected point Q along the x axis, as you can verify by taking the second derivative of Equation 1.23.24Figure 1.16. An object moves in circular motion, casting a shadow on the screen below. Its position at an instant of time is shown.Example 1.5 Circular Motion with Constant Angular SpeedA particle rotates counterclockwise in a circle of radius 3.00m with a constant angular speed of 8.00 rad/s. At t = 0, the particle has an x coordinate of 2.00m and is moving to the right.(A) Determine the x coordinate as a function of time.Solution: Because the amplitude of the particle's motion equals the radius of the circle and ⍰ = 8.00 rad/s, we havex = A cos (⍰t + ⍰) = (3.00 m) cos (8.00 t + ⍰)We can evaluate ⍰ by using the initial condition that x = 2.00 m at t = 0:2.00 m = (3.00 m) cos (0 + ⍰)⍰ + cos–1⍰⍰3.00m⍰2.00 m⍰⍰⍰If we were to take our answer as ⍰ = 48.2° = 0.841 rad, then the coordinate x = (3.00m) cos (8.00t + 0.841) would be decreasing at time t = 0 (that is, moving to the left). 25Because our particle is first moving to the right, we must choose ⍰ = –0.841 rad. The xcoordinate as a function of time is thenx = (3.00 m) cos (8.00t – 0.841)Note that the angle (in the cosine function must be in radians.(B) Find the x components of the particle's velocity and acceleration at any time t.Solution:dxvx⍰dt= (–3.00 m) (8.00 rad/s) sin (8.00t – 0.841)= – (24.0 m/s) sin (8.00t – 0.841)dvax⍰dt= (–24.0 m/s) (8.00 rad/s) cos (8.00t – 0.841)= – (192 m/s2) cos (8.00t – 0.841)From these results, we conclude that vmax = 24.0 m/s and that amax = 192 m/s2.1.5 The PendulumThe simple pendulumis another mechanical system that exhibits periodic motion. It consists of a particle-like bob of mass m suspended by a light string of length L hat is fixed at the upper end, as shown in Figure 1.17. The motion occurs in the vertical plane and is driven by the gravitational force. We shall show that, provided the angle .is small (less than about 10°), the motion is very close to that of a simple harmonic oscillator.The forces acting on the bob are the force T exerted by the string and the gravitational force mg. The tangential component mg sin □ of the gravitational force always acts toward □ = 0, opposite the displacement of the bob from the lowest position. Therefore, the tangential component is a restoring force, and we can apply Newton’s second law for motion in the tangential direction:2Ft □ □mg sin □ □ md s2dtwhere s is the bob’s position measured along the arc and the negative sign indicatesthat the tangential force acts toward the equilibrium (vertical) position. Because s = Land L is constant, this equation reduces to262□d2dt□ □ gsin □LConsidering □ as the position, let us compare this equation to Equation 1.3—does ithave the same mathematical form? The right side is proportional to sin □ rather than to □; hence, we would not expect simple harmonic motion because this expression is not of the form of Equation 1.3. However, if we assume that □ is small, we can use theapproximation sin □ = □; thus, in this approximation, the equation of motion for thesimple pendulum becomes2□d2dt□ □ g□L(for small values of □) (1.24)Now we have an expression that has the same form as Equation 1.3, and we concludethat the motion for small amplitudes of oscillation is simple harmonic motion. Therefore, the function .can be written as □ = □max cos (□t = □), where □max is the maximumangular position and the angular frequency □ is□ □gLThe period of the motion isT □ 2□□ 2□□(1.25)L(1.26)gIn other words, the period and frequency of a simple pendulum depend only on the length of the string and the acceleration due to gravity. Because the period isindependent of the mass, we conclude that all simple pendula that are of equal lengthand are at the same location (so that g is constant) oscillate with the same period. Theanalogy between the motion of a simple pendulum and that of a block–spring system isillustrated in Figure 1.11.The simple pendulum can be used as a timekeeper because its period depends only on its length and the local value of g. It is also a convenient device for making precise measurements of the free-fall acceleration. Such measurements are important because variations in local values of g can provide information on the location of oil and of other valuable underground resources.27Figure 1.17. When □ is small, a simple pendulum oscillates in simple harmonic motion about the equilibrium position □ = 0. The restoring force is –mg sin □, the component of the gravitational force tangent to the arc.Example 1.6 A Connection Between Length and TimeChristian Huygens (1629–1695), the greatest clockmaker in history, suggested that an international unit of length could be defined as the length of a simple pendulum