Part 1 : 1.References Electromagnetics theory [1] Cheng D. ' Fundamental Engineering Electromagnetics' 1st edn.Laser modes Examination of the laser output with a spectrometer of very high resolving power, such as the scanning Fabry-perot interferometer, reveals that it consists of a number of discrete frequency components (or very narrow spectral lines).The photoelectric effect, which is the emission of elwctrons from the surfaces of solids when irradiated, was explanied by Einstein in 1905.He suggested that the energy of a light beam is not spread evenly, but is concentrated certain regions, which propagate like particles.They are simplest form and easy to generate; arbitrary periodic time functions can be expanded into Fourier series of haramonic sinusoidal components; and transient nonperiodic functions can be expreesed as Fourier integeral.High order TEM modes or Hermite-Gaussian modes : We know the usual starting point for the derivation of laser beam propagation modes is solving the scalar Helmholtz equation (wave equation) within the paraxial approximation.However, while the wave theory, as we shell see below, provides an explanation of optical phenomena such as interfererence and diffraction, it fails completely when applied to situation where energy is exchanged, such as in the emission and absorbtion of light.and the photoelectric effect.Field vectors that vary with space coordinates and are sinusoidal fuctions of time can be mathematically represeneted by vector phasers that depend on space coordinates and time as follow 0 E(, , ,) exp ( ) x y z t E i kz t = -?In contrast to this, the the stimulated emission process results in coherent radiation since the wave associated with the stimulating and stimulated photons have identical frequencies, are in phase, have the same state of polarization and travel in the same direction.= = (22) And similarly for the other two components of E and the three components of H. The electric and magnetic fields vibrate perpendicularly to one another and perpendicularly to the direction of propagation as illustrated in Fig.3 that is, light waves are transverse waves.discrete frequency components, they are not concerned with laser propagation where the longitudinal modes all contribute to a single 'spot' of light in the laser output, whereas the transverse modes discussed below may give rise to a pattern of spot in the output.Source of light It is well-known that when an electron in an atom undergoes transitions between energy state or levels it either emits or absorbs a photon, which can be described in term of a wave of frequency ?Theses are : (a) the spontaneous emission process in which the electron drops to the lower level in an entirly random way and (b) the stimulated emission process in which the electron is triggered to undergo the transition by the presence of photons of energy 2 1 ( ) E E- .Let us now examine the frist phasor term on the right side of equation (42) and write 0 ( ) ( ) exp( ) ^ ^ x x x z a E z a E ikz + + E = = - (43) For a cosine reference, the instantaneous expression for E in equation (43) is (,) (,) Re ( )exp( ) ^ ^ x x x x z t a E z t a E z i t ?If such a source is located in an isotropic medium (such as free space) it will radiate uniformly in all directions, the wavefront ate thus a series of concentric spherical shells.B. Transverse modes Longitudinal modes are formed by the plane waves traveiling axaially along the laser cavity on a line joining the centers of the mirror.The fundamental transverse electromagnetic mode (TEM 00) In the case of a fundamental transverse electromagnetic TEM 00 mode, the irradiance distribution across the beam is Gaussian, and so may write the electric field variaition as 2 2 0 2 (,) exp( ) x y E x y w ?Square real part of the amplitude of Hermite-Gauss modes, within the plane z = 0 , for laser beams with beam waist radius 0 w = 4?=10.0m , by discharging an induction coil across a spark gap thereby setting up oscillating electric and magnetic fields. Fig.3 Electromagnetic wave : the electric vector and magnetic vector vibrate in orthogonal planes and perpendicular to the direction of propagation. the remaining two equations (25) and (29) become 2 2 2 0 x x d E k E dz + = (39) 2 2 2 0 y y d H k H dz + = (40) where partial derivative have been replaced by ordinary derivative since H y and Ex are functions of only one variable, z .0 H z + = Thus H y + is the only nonzero component of corresponding H to the E in equation (43), and since 0 ( ) (exp( ) ( ) x x E z E ikz ikE z z z + ?Thus, the laser lineshape will have a finite wavelength (or frquency) spread i.e. thay have a spectral width f d ( ) ?In general, there are many mathemtical details related to this distribution and its properties, but what has been presented here is sufficient to cleaify the spectral lineshape of laser light output. Types of laser spectral lineshape broadenings The spectral lineshape broadening is actually due to a number of external factors and internal atomic processes. Broadened laser transition line (or irradiance against frequency) (a) and (b) cavity modes (c) axial modes in the laser output. For any real laser cavity there will probably be waves traveiling just off axis that are able to replicate themseelve after covering a colsed path such as Fig.9. These will also give rise to resonant modes, but because they have components of their electromagnetic fields which are transverse to the direction of propagation they are termed transverse electromagnetic (or TEM) modes. Visible light and Hertzain waves are part of the electromagnetic spectrum which, as we can see from Table 1, extends approximately over the wave length range of ?Planck's hypothesis did not require that the energy should be emitted in loclazied bundles and it could, with difficulty, be reconcild with the electromagnetic wave theory. For our purposes it is sufficient to accept that in many experiments, especially those invloving the exchange of energy, the particle (photon or quantum) nature of light dominates the wave nature. There is nothing mystical in this, as the electron would undergo this process sooner or later spontaneously : the transition is simply initiated by the presence of stimulating photon. Pure plane wave A. Summary of Maxwell's equations : The results of combing Faraday's law, Ampere's law and Gauss' law are referred to as Maxwell's equations.(p4) C. Time-Harmonic field Maxwell's equations and all the equations derived from them so far in this work hold for electromagentic quantities with an arbitrary time dependance.(30) D. Uniform plane wave A uniform plane wave is a particular solution of Maxwell's equations with E (and H) assumeing the same direction, same magnitude, and same phase in infinite planes perendiduclar to the direction of propgation. E (45) It is clear that quation (45) represent a traveling wave and descibes a perfectly monochromatic plane wave of infinite extent propgation in the postive z direction.= = (47) We can see, the second phasor term on the right side of equation (42) 0 E ikz exp( ) − − , represents a cosinusoidal wave traveling in the (−z) direction with the same velocity ?This can be simply seen practically in both emission and absorption processes and if, for example, we were to measure the transmission (or emission) as a function of frquency for transition between the energy states E1 and E2 , we would obtain a probabilty distribution. Changing u correponds to moveing the curve to another postion (translating it), and for u = 0 it is symmetric with respect to the ordinate (i.e. verticle direction), as shown in Fig.6. A. Longitudinal modes The two mirror of the laser form a resonant cavity and standing wave patterns are set up between the mirror in exactly the same way that standing waves develop on the string. The modes of oscillation of the laser cavity will consist, therefore of a large number of frequencies, each given by equation (73) and separated by c L / 2 , as shown in Fig.8.It should be appreiated, however, that while all the integers n give possible axial cavity modes only those which lie within the gain curve or laser transition line will actually oscillate.Fig.10 shows the typical variation of w , with position, within a cavity formed by two concave mirrors of radius of curvature 1 r and 2 r separated by L .Nature of light During the seventeenth century two emission theories on the Nature of light were developed, the wave theory of Hooke and Huygens and the corpuscular theory of Newten.Then in 1864 Maxwell combined the equation of electromagnetism in a general form and showed that they suggest the existance of trensverse electromagnetic wave. Maxwell theory suggested the posibilty of producing electromagnetic waves with a wide range of frequencies (or wavelengths).In 1887 Hertz succeeded in generateing non-visible electromagnetic waves, with a wavelength of the order of ?As the absorption transition, in common with stimulated emission, can only occur in the presence of photon of appropriate energy, it is often referred to as stimulated absorption.D E H J (11) B. Waves equations : First we derive the wave equation that governs the propagation of all electromagnetic wave.Each equation is composed of three scalar differential equations in term of the components of the vectors.This simplified diagrams and mathematical descriptions but we should always remember that there is also a magnetic field component which behaves in similar way to the electric field component.We may arbitrarity assume the direction of E to be in the postive x direction; that is ( ) ^ E = E z a x x (31) This x component of E is a function of only z since the field is to be uniform over the xy plane x and y is thus independent of and coordinates.( ) ( ) ( ) E z E z E z x x x + − = + (41) 0 0 ( ) exp( ) exp( ) E z E ikz E ikz x + − = − + − (42) where E0 + and E0 − are arbitrary constants that must be dertemined by boundary condition.Thus if we fix our attantion on a particular point (a point of particular phase) on the wave, we set cos( ) tan ?t kz a cons t − = or ?t kz A cons t phase − = tan , from which we obtain 0 0 1 c k ?Thus, for example have plane wave propagating in direction yz to the z axis with its wavefront normal to the yz plane, we can write ^ ^ y y z z k = + k a k a (57) ^ ^ y z r = + ya za (58) .Laser line shapes In deriving the expression for the propagation of plane wave acuually represents the ideal case.It is implicity assumes that all the atoms in either the upper or lower levels would be able to interact with the perfectly monchromatic wave with lineshape 0 f ( ) ?Although, the spectral width of a laser output can be much less than that of ordinary light due to the spontaneous emission process, it cannot really be considered monchromatic wave.It is considered the most important continous distribution because in applications many randam variables are normal randam variables, (that is, they have a normal distribution) or they are approximately normal or can be transformed into normal randam variables in relatively simple fashion.Furthermore, the normal distribution is a useful approximation of more complicated distributions, and it also occurs in the proofs of various statistical tests.The normal distribution, also known in physics studies as Gaussian distribution, is defined as the distribution with density (or the probabilty density function) 2 1 1 ( ) exp , (0) 2 2 x f x u ?It is alos a useful approximation of more complicated distributions, and it also occurs in the proofs of various statistical tests.The processes involved may be : (1) collision or (2) electromagnetic or (3) just the uncertainty broadening associated with the spontaneous lifetime.is the linewidth (full-width half maximum), that is the separation between the two points on the (frquency) curve where the function falls to half of its peak value which occurs at frquency ?Practically, the inhomogeneous broadening mechanisms lead to a Gaussian lineshape which may be written in terms of frequency as 2 0 1 1 ( ) exp 2 2 G f ?= − = + (74) As equation (74) is independent of n , the frequency separation of adjacent modes is the same irrespective of their actual frequencies.This is not accident but merely a direct consequence of the requirement that the mode be self replication as the light energy flows backwards and forwards between mirrors .Subsequent observations by Young, Malus, Euler and ithers lent support to the wave theory.On the other hand, for experiments invloving interfererence and diffraction, where light interacts with light, the wave nature dominates.Let us consider the electron transitions which may occur between the two energy level s of the hypothetical atomic system shown in Fig 1.Under normal circustances we do not observe the stimulated emission process because the probabilty of the the spontaneous emission ?Because spontaneous radiation from any atom is emitted at random, the radiation emitted by a large number of atoms will clearly be incoherent.Energy level diagram illustrating (a) absorption, (b) spontaneous emission and (c) stimulated emission. Associated with Maxwell's equations, we have equation of continity (or conservation of charge) t ??x ?x = − E E (16) We similarly obtain, by taking the curl of equation (13) and substituting equation (12), 2 0 0 2 ( ) dt u ?In describing optical phenomena we often omit the magnetic field vector.(24) where E(, , ,) x y z t and H(, , ,) x y z t are the value of the electric and magentic fields at the point r at time t , E0 and H0 are the amplitudes of the electric and magnetic waves, ?(26) Expanding equations (25) and (26) in terms of components, the wave equations for the phasor components of the field vector become 2 2 2 2 2 2 2 x x x x E E E k E x y z ?Stricly speaking a uniform plane wave does nor exist in practical because a sources infinite in extent would be required to create it, and practical wave sources are alawyes finite in extent.Here we should mentioned to that, equation (45) can also be expressed using a sine rather that a cosine function, or alterntively using complex expoentials.u = = = (46) Equation (46) assures us that the the velocity of propagation of a equiphase fron (the phase velocity) is equal to the velocity of light.As we know, it is impossible in practice to produce perfectly monochromatic waves, we often have the situation where a group of wave of closely similar wavelength is moving such that their resultant forms a packet.y z k r = + k y k z (59) By follow the same analysis of one dimensional plane wave, hence we can write equation (56) in this case as 0 (, , ,) cos( ) y z E x y z t E t k y k z = − − ?) ensity energy time area = is proportional to the square of the amplitude, there is an inverse-square-law decrease in irradiance.Likewise, it is a good check for robust techniques that are designed to work well under a wide variety of distributional assumptions.Practically, the homogenous broadening mechanisms lead to a Lorentzian lineshape which may be written in terms of frequency as ( ) 2 2 0 (/ 2) ( ) 2 L f ?In order to comply with what is required in laser light because the broadening in it occurs to the frequency (or wavelength).In some lasers books, this lineshape is called Doppler frequency distribution because it is source of inhomogeneous broadening.In order to comply with what is required in laser light because the broadening in it occurs to the frequency (or wavelength).In this case electric field distributions are essentially given by the product of a Gaussian function and a Hermite polynomial, apart from the phase term as follows : ( ) 2 0 0 2 2 2 2 2 (, ,) .and u0 yielded a vaule for c in very close agreement with the value of the speed of light in vacuo measured independently.Maxwell therefore proposed that light was an electromagnetic wave having a speed of 8 c m s = x3 10 / , a frequency of some 14 f Hz = x5 10 and a wavelength of about ?Planck found that the observation indicated that light energy is emitted in muliplies of certain minimum enregy unit.When Einstein showed, however, that it seems necessary asssume the concentration of energy traveling through space as particles, a wave solution was excluded.If the electron is in the lower E1 then in the presence of photons of enegy 2 1 ( ) E E− it may be excited to the upper level E2 by absorbing a photon.Alternatively if the electron in the level E2 it may return to the ground stste with the emission of a photon.Two energy level system The absorption and emission processes are illustrated in Fig 2.(a), (b) and (c).This means that with stimulated emission the amplitude of an incident wave can grow as it passes through a collection of excited atom in what is clearly an amplification process.Propagation of light Propagation of light refers to the manner which is an electromagntic wave trsnsfer it's energy from one point ot another.This maen that, we will derive the main equation described the propagation of light in empty space.It is known that the first approximation represents the ideal situation and proceed directly from Maxwell's equations.Although this approximation does not represent the practical cases, such as Gaussian wave and so on , it constitutes the theoretical basis for their derivation.Although these equations have not been derived analytically, the are reasonable and no experiments have shown them to be invalid.In the absence of any such data, we may accept them as a valid characterization of electromagnetic phenomena.= .H 0 (15) Taking the curl of equation (12) and substituting equation (13), we obtain 2 0 0 2 ( ) dt u ?In rectangular coordinates, the vector Laplancian is given by 2 2 2 2 2 2 2 A A x z A y x y z ?A (19) Substituting equation (18) into equations (16) and (17), we obtain 2 2 0 0 2 dt u ?But if we are far enough away from a source, the wavefront beacomes almost sperical; and very small portion of the surface of a giant sphere is very nearly a plane.The characteristics uniform plane waves are particular simple, and their study is of fundamental theoretical as well as practical importance.At successive time the curve effectively travels in the postive z direction.= 500nm .=100.0km .??2.3.?(1) ?????= .( ) ????????????????− ?( ) (.???= + + ?????????? ?/).?(25) 2 0 0 ??????????????????(28) ???????????= = ???= ?+ + = = ????????????.????.+ + + + ??????????????+ + ?= ??????= = = = ?+ + = = ???????= ?(55) ????.??.??+ .??????????????????2 ???????.????????????(63) ???=1/ (2) .?= ????− ??+ ????????????= ?(67) ????????= ???????− + ???????= ?.???= ???(69) ??????????????????????= ?.???= ????1 ?= ?(72) 2.= = ???3.?????= + ??????????4.????????????????????= ?= ?− ?= ?− ?= ?− ?.??