The aim of this chapter is to introduce axiomatically the set of Real numbers.W. Rudin Principles of Mathematical Analysis.Z+ with ba. In order to make this equation soluble we have to enlarge the set Z+ by introducing negative integers as unique solutions of the equations a + x = 0 (existence of additive inverse) 28 CHAPTER 2.N. This necessitates adding {0} to N, declaring 01, thereby obtaining the set of non−negative integers Z+.(Fundamental theorem of arithmetic) Every positive integer except 1 can be expressed uniquely as a product of primes.N. Then (a b > c d) (adbc) . The following theorem provides a very important property of rationals. Theorem 2.1.2. Between any two rational numbers there is another (and, hence, infinitely many others).Z. In order to solve (2.1.1) (for a = 0) we have to enlarge our system of numbers again so that it includes fractions b/a (existence of multiplicative inverse in Z − {0}).Those of you familiar with basic concepts of algebra will find that axioms A.1 − A.11 charac terize R as an algebraic field.N. Our extended system, which is denoted by Z, now contains all integers and can be arranged in order Z = {.Hence, appealing to the Fundamental Theorem of Arithmetic, p 2 is even, and hence p is even.Multiplying this equality by n 2 we obtain m3 n = mn + 7n 2 , which is impossible since the right−hand side is an integer and the left−hand side is not.2.2 The Field of Real Numbers In the previous sections we discussed the need to extend N to Z, and Z to Q. The rigorous construction of N can be found in a standard course on Set Theory.The last axiom links the operations of summation and multiplication. The set of rationals Q also forms an algebraic field (that is, the rational numbers satisfy axioms A.1 − A.11).(order) The first difficulty occurs when we try to come up with the additive analogue of a.1 = 1.a = a for a ?Indeed, since b, d and m are positive we have [a(b + md)b(a + mc)] [madmbc] (adbc), and [d(a + mc)c(b + md)] (adbc).Suppose for a contradiction that the rational number p q (p ? Z, q ? N, in lowest terms) is such that (p q) 2 = 2.In this course we postulate the existence of the set of real numbers R as well as basic properties summarized in a collection of axioms.R) [(ab)c = a(bc)] (associativity of multiplication).We also mention at this point the Fundamental theorem of arithmetic.Z. The equation (2.1.1) ax = b need not have a solution x ?Here hcf(p, q) stands for the highest common factor of p and q, so when writing p/q for a rational we often assume that the numbers p and q have no common factor greater than 1.All the arithmetical operations in Q are straightforward.Q and consider the equation (2.1.2) x 2 = a. In general (2.1.2) does not have rational solutions.The last statement contradicts our assumption that p and q have no common factor.The last theorem provides an example of a number which is not rational.No rational x satisfies the equation x 3 = x + 7.First we show that there are no integers satisfying the equation x 3 = x + 7.For a contradiction suppose that there is. Then x(x + 1)(x − 1) = 7 from which it follows that x divides 7.Direct verification shows that these numbers do not satisfy the equation.Second, show that there are no fractions satisfying the equation x 3 = x+7.Theorem 2.1.1..1 = 1 ....{0} ?!!!!