

A perspective on the foundations of decimal numeration In fact, in our decimal numeration system, it is possible to have different representations for the same number. This arises from the fact that some numbers have an infinite recurring decimal representation, as is the case with $0.9999\dots$, and a terminating representation, such as $1.0000\dots$ This demonstrates a fundamental property of decimal numeration: a number can be represented in multiple equivalent ways. The equality $0.9999\dots = 1$ is an illustration of this property. As you know, we know that every real number can be expressed in decimal form, that is, by providing a sequence of digits before and after the decimal point. I have already had the opportunity to discuss this notation in my post about universal numbers. For the simplest numbers, this sequence is finite. For others, the sequence of decimals may be infinite but repeating periodically, for example: $22/7 = 3.142857142857142857142857\dots$ And finally, for the most capricious numbers, this sequence is infinite and without any particular periodicity. Today we will focus on a simple periodicity: a single digit that repeats.