3 Sum and direct sum of Ideals In this section we discuss the concept of the sum of ideals (right ideals, left ideals) in a ring R. Definition...,R,, be afamily of rings, and let R = R1 X R2 X" X R,, be their direct product. Let R? E R,). Then R -- is a direct sum of ideals R?, and R? R, as rings. On the other hand, fR = (R)17...IA,, adirect sum of ideals ofR, then R A1 X A2 X X A,,, the direct product oftheA,'s considered as rings on their own right. Proof Clearly, R R a R. Let xERrfl j*, Then x=(0,0,...,a,,0,...,0)(a1,a2,...,a,_1,0,a,+1,...,a,J. This gives a, = 0 and, hence, x =0. Therefore, 198 Ideals and bomoinorpldsns For the second part we note that ifxE R, then x can be uniquely expressed asa1+a2++ a,a,EA,, by Because and onto. Also, if x,y isdirect,f is well defined. It is also clearthatf is both 1-1 (R)17..IA,, thenf(x + y) =f(x) +f(y). To show we need to note that if a1 + since a,b, E A, fl 4 (0). This remark then immediately yields that f(xy) =f(x)f(y). Hence,f is an isomorphism. 0 Thedirect sumR -- (R) 17..1A,is also call ed the (internal) direct swn of idealsA,inR, (external) direct sum of the family of rings A,, i -- 1,2,...,n. In the tatter case the notation A1 (R)A2 0... 0 A,, is also frequently used. The context will make clear the sense in which the term "direct sum" is used.If A,,A2 A,, are right ideals in a ring R, then S(a1 +a2+ +a,,Ia,EA,, i= 1,2 n) is the of right ideals A1,A2 A,, Proof It is clear that (a, + a2 + + a,ja, E A,, i= 1,...,n) isa right ideal in R. Also, if a,EA,, then a1=a1 in S, and, hence, A, C S. Similarly, each A,, i 2,...,n, is contained in S. Further, if Tis any right ideal in R containing each A,, then obviouslyT S. Thus, S is the intersection of all the right ideals in R containing each A,.LeIA, ,A2,....A,, beafa,nilyofright ideals in a ringR.is a minimal right ideal, then I is generated by any nonzero element of!.3.2 Theorem.LetA1 right (or left) ideals inaringR.